Math 215 Winter 2022

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Office hours : Mon, Fri 10-11 am in 4088 East Hall

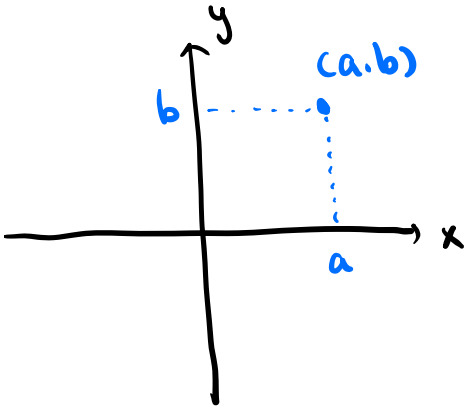
Tue 1-2 pm in 4f23 East Hall

Syllabus on the Canvas page.

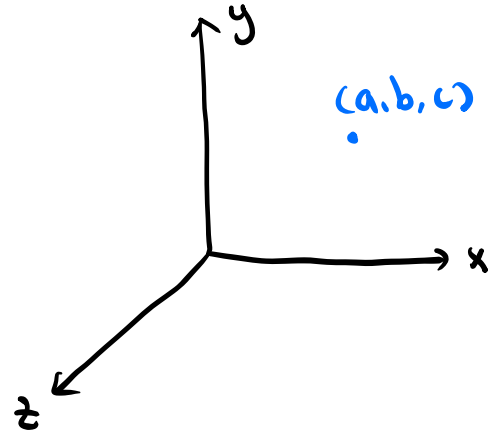
12.1. Coordinate systems

Course theme : We study multivariable functions and 3-dimensional coordinate systems using our knowledge on single-variable calculus and 2-dimensional coordinate systems

Def The standard (rectangular) coordinate systems are defined by perpendicular axes.



2-dim'l system $\mathbb{R}^2$



3-dim'l system $\mathbb{R}^3$.

Thm (Distance formula) = Pythagorean theorem

(1) Distance between (x_1, y_1) and (x_2, y_2) on $\mathbb{R}^2$

is $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

(2) Distance between (x_1, y_1, z_1) and (x_2, y_2, z_2) on $\mathbb{R}^3$

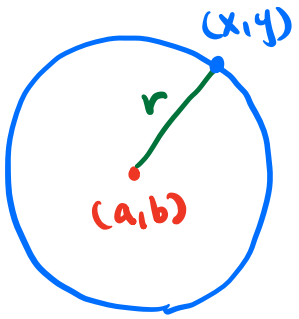
is $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$

★ **Ex** (1) Find an equation of the circle in $\mathbb{R}^2$ of radius r and center (a, b).

Sol radius = distance from center

$\rightsquigarrow r = \sqrt{(x-a)^2 + (y-b)^2}$

$\rightsquigarrow \boxed{r^2 = (x-a)^2 + (y-b)^2}$

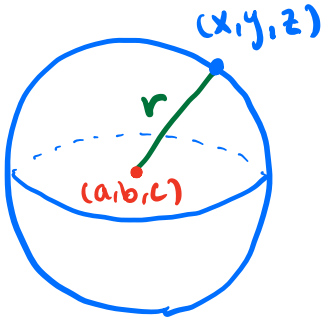


(2) Find an equation of the sphere in $\mathbb{R}^3$ of radius r and center (a, b, c).

Sol radius = distance from center

$\rightsquigarrow r = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$

$\rightsquigarrow \boxed{r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2}$



Ex Sketch the graph of $z = x^2 + y^2$ ← Very important surface

Sol Tips for sketching graphs:

(1) Look at "cross sections" by setting one of the variables to be constant.

(2) It's often convenient to put the output variable on the vertical axis.

Output variable in this case is z

(i.e. z is given by a function of x and y)

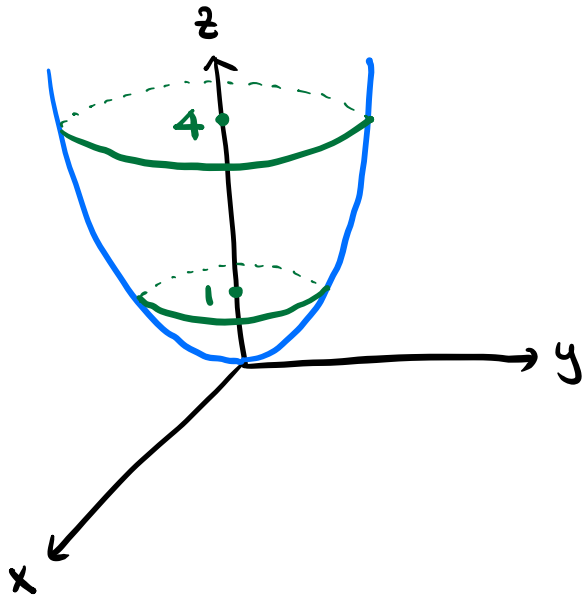
Set $x = 0$: $z = y^2$ ⇝ a parabola

(yz-plane)

Set $z = 0$: $0 = x^2 + y^2$ ⇝ $x = y = 0$ ⇝ a point.

Set $z = 1$: $1 = x^2 + y^2$ ⇝ a circle of radius 1.

Set $z = 4$: $4 = x^2 + y^2$ ⇝ a circle of radius 2.



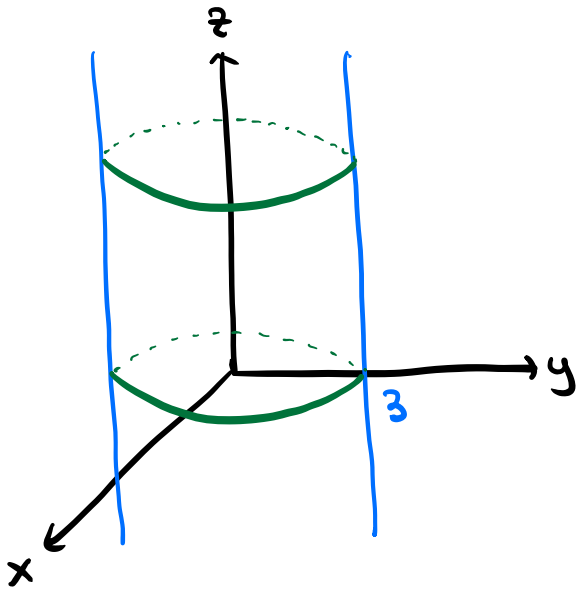
* This surface is called a (circular) paraboloid.

Ex (1) Sketch the surface $x^2+y^2=9$.

Sol Set $z=0$: $x^2+y^2=9 \rightsquigarrow$ a circle of radius 3

Set $z=1$: $x^2+y^2=9 \rightsquigarrow$ a circle of radius 3

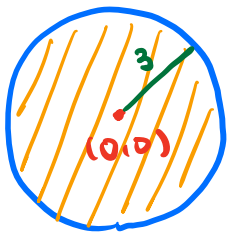
$\vdots$



* This surface is a circular cylinder of radius 3 along the z-axis.

(2) Describe the region $x^2+y^2 \le 9$.

Sol For each z, the cross section is given by $x^2+y^2 \le 9$, which represents the disk of radius 3 and center $(0,0)$



$\left(\because x^2+y^2 \le 9 \rightsquigarrow \sqrt{x^2+y^2} \le 3 \right.$

$\left. \Rightarrow \text{distance from } (0,0) \text{ is at most } 3 \right)$

So the region $x^2+y^2 \le 9$ in $\mathbb{R}^3$ is

the solid cylinder of radius 3 along the z-axis (surface + inside)